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Reducing PAPR of OFDM Signal using PTS and COPTS

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***Abstract* — Electronics communication system has revolutionized the face of the world. Communication with someone a mere century back was only possible by physical mode. But now that can be done just by clicking a switch on the telephone pad or by just a click of the mouse. Even live television report, live games telecast could not be possible without wireless communication. Communication is one of the important aspects of life. With the advancement in age and its growing demands, there has been rapid growth in the field of communications. Signals, which were initially sent in the analog domain, are being sent more and more in the digital domain these days. For better transmission, even single carrier waves are being replaced by multi carriers. Multicarrier systems like CDMA and OFDM are nowadays being implemented commonly. OFDM system is vastly used in communication system, but there is major problem faces when implement the system is high peak to average power ratio of this system. There are number of techniques to deal with the problem of PAPR. Some of them are partial transmit sequence (PTS) and Complexity PTS. In this paper we have focused on learning the basics of an OFDM System and have undertaken various methods to reduce the PAPR in the system so that this system can be used more commonly and effectively.**

***Keywords* – OFDM, FFT, ISI, ICI, Modulation and Demodulation, PAPR, CCDF, PTS and Reduced Complexity PTS (COPTS).**

I. INTRODUCTION

Since the very genesis of man, communication has been one of the main aspects in human life. Previously various methods like sign languages were implemented for this purpose. As various civilizations started coming into existence, many innovative ideas came to the minds of the people special birds and human messengers were employed to meet these challenges. As ages rolled by, post system developed and transportation vehicles like trains and ships were used to maintain link between people miles apart. But by the turn of the nineteenth century, a great leap in communication system was observed when wireless communication was introduced. After the advent of wireless communication huge change has been observed in the lifestyle of people. Wireless communication which was initially implemented analog domain for transfer has is nowadays mostly done in digital domain. Instead of a single carrier in the system multiple sub-carriers are implemented to make the process easier. Orthogonal Frequency Division Multiplexing is a special form of multicarrier modulation which is particularly suited for transmission over a dispersive channel. Here the different carriers are orthogonal to each other, that is, they are totally independent of one another. This is achieved by placing the carrier exactly at the nulls in the modulation spectra of each other.

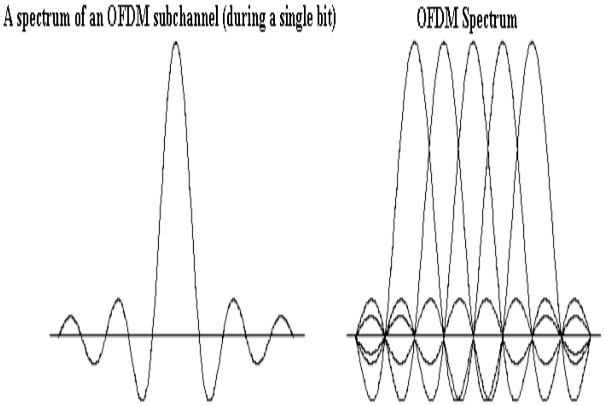

Fig.1. OFDM Spectrum.

Two periodic signals are orthogonal when the integral of their product over one period is equal to zero.
For the case of continuous time:

$$\int_0^T \cos\left(2\pi n f_0 t\right) cos\left(2\pi m f_0 t\right) \mathrm{dt} = 0,$$

For the case of discrete time:

$$\sum_{k=0}^{N-1} \cos\left(\frac{2\pi \mathrm{kn}}{\mathrm{N}}\right) \cos\left(\frac{2\pi \mathrm{kn}}{\mathrm{N}}\right) \mathrm{dt} = 0,$$

where $m \neq n$ in both cases.

OFDM is one of the many multicarrier modulation techniques, which provides high spectral efficiency, low implementation complexity, less vulnerability to echoes and non-linear distortion. Due to these advantages of the OFDM system, it is vastly used in various communication systems. But the major problem one faces while implementing this system is the high peak to average power ratio of this system. A large PAPR increases the complexity of the analog to digital and digital to analog converter and reduces the efficiency of the radio frequency (RF) power amplifier. Regulatory and application constraints can be implemented to reduce the peak transmitted power which in turn reduces the range of multi carrier transmission. This leads to the prevention of spectral growth and the transmitter power amplifier is no longer confined to linear region in which it should operate. This has a harmful effect on the battery lifetime. Thus in communication system, it is observed that all the potential benefits of multi carrier transmission can be out - weighed by a high PAPR value. There are a number of techniques to deal with the problem of PAPR. Some of them are “amplitude clipping”, “clipping and filtering”, “coding”, “partial transmit sequence (PTS)”, selected mapping (SLM) and “interleaving”. These techniques achieve PAPR reduction at the expense of transmit signal power increase, bit error rate (BER) increase, data rate loss, and computational complexity increase.

II. PEAK TO AVERAGE POWER RATIO

Presence of large number of independently modulated sub-carriers in an OFDM system the peak value of the system can be very high as compared to the average of the whole system. This ratio of the peak to average power value is termed as Peak-to-Average Power Ratio. Coherent addition of N signals of same phase produces a peak which is N times the average signal. The major disadvantages of a high PAPR are increased complexity in the analog to digital and digital to analog converter and reduction is efficiency of RF amplifiers.

Let the data block of length N be represented by a vector. Duration of any symbol in the set **X** = $[X_o, X_o, \dots, X_{N-1}]^T$ is T and represents one of the subcarriers $\{f_n, n = 0, 1, \dots, N-1\}$ set. As the N subcarriers chosen to transmit the signal are orthogonal to each other, so we can have $f_n = n\Delta f$, where $n\Delta f = 1/NT$ and NT is the duration of the OFDM data block **X**. The complex data block for the OFDM signal to be transmitted is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n, e^{j2\pi n\Delta f t}, \qquad 0 \le t \le NT,$$

The PAPR of the transmitted signal is defined as

$$\text{PAPR} = \frac{\max_{0 \le t \le NT} |x(t)|^2}{1/NT \int_0^{NT} |x(t)|^2 dt}$$

Reducing the *max*|*x (t)*| is the principle goal of PARP reduction techniques. Since, discrete – time signals are dealt with in most systems, many PAPR techniques are implemented to deal with amplitudes of various samples of x (t). Due to symbol spaced output in the first equation we find some of the peaks missing which can be compensated by oversampling the equation by some factor to give the true PAPR value.

III. CUMULATIVE DISTRIBUTION FUNCTION

Cumulative Distribution Function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of any PAPR technique. Normally, the Complementary CDF (CCDF) is used instead of CDF, which helps us to measure the probability that the PAPR of a certain data block exceeds the given threshold. By implementing the Central Limit Theorem for a multicarrier signal with a large number of subcarriers, the real and imaginary part of the time domain signals have a mean of zero and a variance of 0.5 and follow a Gaussian distribution. So Rayleigh distribution is followed for the amplitude of the multicarrier signal, where as a central chi-square distribution with two degrees of freedom is followed for the power distribution of the system.

The CDF of the amplitude of signal sample is given by

$$F(z) = 1 - \exp(z)$$

The CCDF of the PAPR of the data block is desired is our case to compare outputs of various reduction techniques. This is given by

$$\begin{aligned} P(PAPR > z) &= 1 - P(PAPR \le z) \\ &= 1 - F(z)^n \\ &= 1 - (1 - \exp(-z))^n \end{aligned}$$

IV. PAPR REDUCTION TECHNIQUES

PAPR reduction techniques vary according to the needs of the system and are dependent on various factors. PAPR reduction capacity, increase in power in transmit signal, loss in data rate, complexity of computation and increase in the bit-error rate at the receiver end are various factors which are taken into account before adopting a PAPR reduction technique of the system. The PAPR reduction techniques on which we would work upon and compare in our later stages are as follows:

A. Amplitude Clipping and Filtering

Amplitude clipping is considered as the simplest technique which may be under taken for PAPR reduction in an OFDM system **[8]**. A threshold value of the amplitude is set in this case to limit the peak envelope of the input signal. Signal having values higher than this predetermined value are clipped and the rest are allowed to pass through un-disturbed.

$$B(x) = \begin{cases} x, & |x| \le A \\ Ae^{j\varphi(x)}, & |x| \ge A \end{cases}$$

where,

B(x) = the amplitude value after clipping.

x = the initial signal value.

A = the threshold set by the user for clipping the signal.

The problem in this case is that due to amplitude clipping distortion is observed in the system which can be viewed as another source of noise. This distortion falls in both in band and out of band. Filtering cannot be implemented to reduce the in band distortion and an error performance degradation is observed here. On the other hand spectral efficiency is hampered by out of band radiation. Out of band radiation can be reduced by filtering after clipping but this may result in some peak regrowth. A repeated filtering and clipping operation can be implemented to solve this problem. The desired amplitude level is only achieved after several iteration of this process.

B. Selected Mapping

The main objective of this technique is to generate a set of data blocks at the transmitter end which represent the original information and then to choose the most favorable block among them for transmission **[9]**. Let us consider an OFDM system with N orthogonal subcarriers. A data block is a vector $X = (x_n)_N$ composed of N complex symbolsx_n, each of them representing modulation symbol transmitted over a subcarrier X is multiplied element by element with U vector $B_u = (b_{u,n})_N$ composed of N complex number $b_{u,n}$, u ϵ {0,1,…,U – 1} defined so that $|b_{u,n}| = 1$, where |.| denotes the modulus operator. Each resulting vector $X_u = (x_{u,n})_N$, where $x_{u,n} = b_{u,n} \cdot x_n$, produces after IDFT, a corresponding OFDM signal $s_u(t)$ given by

$$s_u(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_{u,n} e^{j2\pi n\Delta f t}, \qquad 0 \le t \le T,$$

Where T is the OFDM signal duration and $\Delta f = 1/T$ is the subcarrier spacing.

Among the modified data blocks, the one with the lowest PAPR is selected for transmission. The amount of

PAPR reduction for SLM depends on the number of phase sequences *U* and the design of the phase sequences.

C. Partial Transmit Sequence

In the PTS technique, input data block X is partitioned in *M* disjoint sub blocks $X_m = [X_{m,0}, X_{m,1}, \dots, X_{m,N-1}]^T$, m = 1, 2, … , M, such that $\sum_{m-1}^{M} X_m = X$ and the sub – blocks are combined to minimize the PAPR in the time domain. The *L* times oversampled time domain signal of X_m, m = 1, 2, …. , M, is obtained by taking the IDFT of length NL on X_m concatenated with *(L – 1) N* zeros. These are called the partial transmit sequences. Complex phase factors, $b_m = e^{j\varphi_m}, m = 1, 2, \dots, M,$ are introduced to combine the PTSs. The set of phase factors is denoted a vector b = $[\, b_1, b_2, \dots, b_M]^T$. The time domain signal after combining is given by

$$x'(b) = \sum_{m=1}^{M} b_m . x_m$$

where,
$x'(b) = [x'_0(b), x'_1(b), \dots, x'_{NL-1}(b)]^T$.

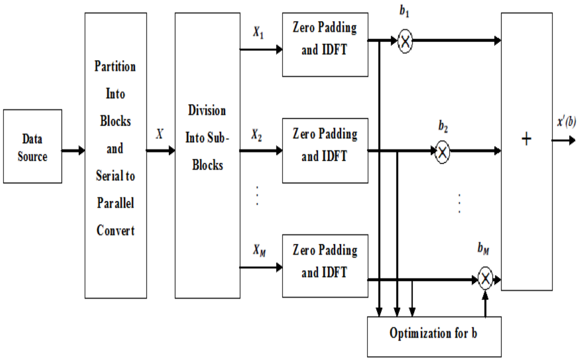

Fig.2. Block Diagram of PTS techniques.

The objective is to find the set of phase factors that minimizes the PAPR. Minimization of PAPR is related to the minimization of $\max_{0 \le k \le NL-1} | x'_{NL-1}(b)|$.

D. Complexity Reduced Partial Transmit Sequence

Although the iterative flipping algorithm significantly reduces the search complexity, there is some performance gap between the ordinary PTS technique and the iterative flipping algorithm. The iterative flipping algorithm changes the phase factors one by one and simply in the order of increasing *m*. If we find better order for changing the phase factors, we might expect improved performance. In addition, if we increase the number of phase factors explored simultaneously, further improvement can be expected. Here, we propose a novel PTS technique based on gradient descent search which is useful in solving combinatorial optimization problems. The objective of the technique is to find the phase factors that achieve PAPR statistic close to that of the ordinary PTS technique with reduced search complexity and little performance degradation [15]. The proposed technique starts with a pre-determined vector of phase factors. Next, it finds an updated vector of phase factors in its "neighborhood" that results in the largest reduction in PAPR. Neighborhood of radius is defined as the set of vectors with Hamming distance equal to or less than from its origin. The equation that updates the vector of phase factors from *b* to b' is given by

$$b' = \arg\left\{\max_{\|\hat{b}-b\|_{H \le r}} (PAPR\ For\ b - PAPR\ For\ \hat{b})\right\}$$

where $\|.\|_H$ denotes the Hamming weight of its vector argument and denotes the radius of the neighborhood which is centered at *b*. This process is repeated using the updated vector of phase factors as a new starting point as long as PAPR reduction is achieved. We may limit the number of maximum iterations to update the phase factors. The performance and complexity of the technique is dependent on the value of r. If r is equal to M, the proposed technique searches for all combinations of allowed phase factors. In this case, the proposed technique is equivalent to the ordinary PTS technique. Hence, the proposed technique includes the ordinary PTS technique as a special case. If *r = 1*, it offers search complexity which grows linearly with the number of subblocks *M*. Various *r* between 1 and *M* correspond to algorithms which provide tradeoffs between performance and complexity.

V. EXPERIMENTAL RESULTS AND ANALYSIS

OFDM is one of the many multicarrier modulation techniques, which provides high spectral efficiency, low implementation complexity, less vulnerability to echoes and nonlinear distortion. Due to these advantages of the OFDM system, it is vastly used in various communication systems. But the major problem one faces while implementing this system is the high peak to average power ratio of this system. A large PAPR increases the complexity of the analog to digital and digital to analog converter and reduces the efficiency of the radio frequency (RF) power amplifier. Regulatory and application constraints can be implemented to reduce the peak transmitted power which in turn reduces the range of multi carrier transmission. This leads to the prevention of spectral growth and the transmitter power amplifier is no longer confined to linear region in which it should operate. This has a harmful effect on the battery lifetime. Thus in communication system, it is observed that all the potential benefits of multi carrier transmission can be outweighed by a high PAPR value. There are a number of techniques to deal with the problem of PAPR. Some of them are "amplitude clipping", "clipping and filtering", "coding", "partial transmit sequence (PTS)", "selected mapping (SLM)" and "interleaving". These techniques achieve PAPR reduction at the expense of transmits signal power increase, bit error rate (BER) increase, data rate loss, computational complexity increase.

We know that Orthogonal Frequency Division Multiplexing (OFDM) is very attractive technique for multicarrier transmission and has become one of the standard choices for high speed data transmission over a communication channel. It has various advantages, but one major drawback: it has a very high PAPR. It is necessary to reduce the PAPR value by using different techniques so firstly to plot the Bit Error Rate (BER) against Signal to

Noise Ratio (SNR) to understand the OFDM system. To reduce Peak Average Power Ratio (PAPR) value according to Complementary Cumulative Distribution Function (CCDF) by using Partial Transmit Sequence (PTS) and reduced complexity PTS (COPTS). At last gives comparison curve of PTS and COPTS on the basis of PAPR value and BER also. All simulation done on Matrix Laboratory (MATLAB) simulator. Figure 3 shows the comparison curve of bit error rate against signal to noise ratio of an OFDM System after and before applies PTS and COPTS.

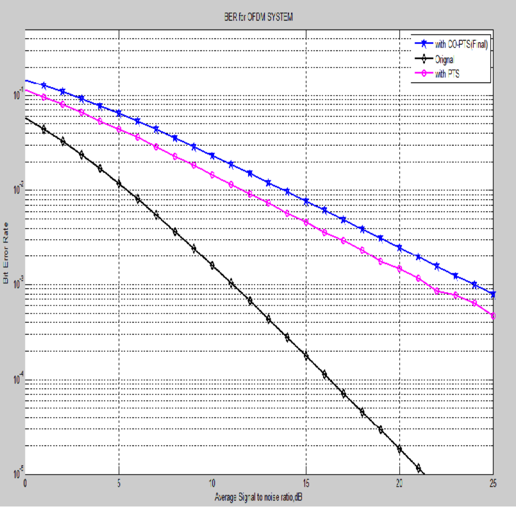

Fig.3. BER vs. SNR plot of an OFDM System after and before applies PTS and COPTS.

So we find out the bit error rate (BER) according to signal to noise ratio (SNR) of an orthogonal frequency division multiplexing (OFDM) after and before applies partial transmit sequence and complexity partial transmit sequence PTS and COPTS respectively. In Figure 3 it is clearly seen that bit error rate against signal to noise ratio is increased by using different techniques such as base paper technique, partial transmit sequence and reduced complexity partial transmit sequence. At particular point (it's 5dB), to detect the bit error rate value which is 0.011, 0.014 and 0.025 by using ordinary technique, partial transmit sequence and reduced complexity sequence respectively.

After that Figure 4 show the curve of peak average to power ratio (PAPR) reduction according to ccdf measurement for an OFDM system. To calculate the PAPR of an OFDM Signal according to complementary cumulative distribution function (ccdf) measurement with partial transmits sequence (PTS) technique. It shows the PAPR value according to complementary cumulative distribution function (ccdf) measurement by using partial transmit sequence PTS technique and its value vary from 7 to 8 which is shown in the Figure 4.

After that Figure 5 show the curve of improved peak average to power ratio (PAPR) reduction according to ccdf measurement for an OFDM system. To calculate the PAPR of an OFDM Signal according to complementary cumulative distribution function (ccdf) measurement with reduced complexity partial transmits sequence (COPTS) technique. It shows the PAPR value according to complementary cumulative distribution function (ccdf) measurement by using reduced complexity partial transmit sequence COPTS technique and its value vary from 5 to 6 which is shown in the Figure 5.

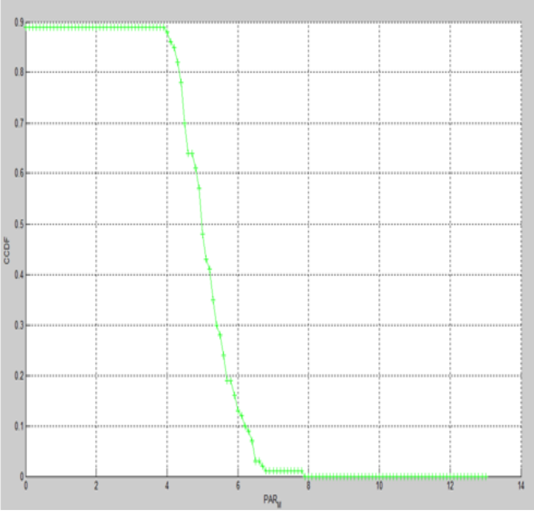

Fig.4. Plot PAPR Reduction according to ccdf measurement for an OFDM system.

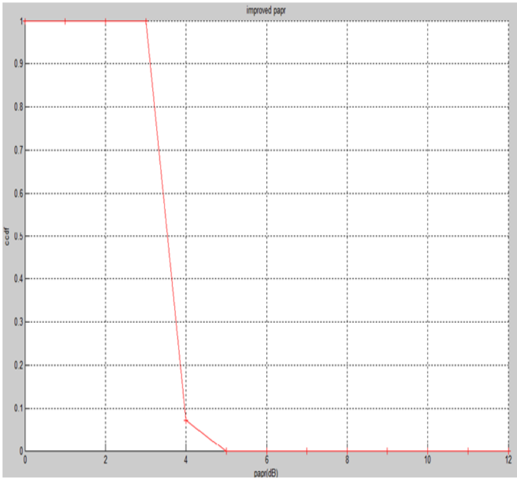

Fig.5. Plot improved PAPR Reduction according to ccdf measurement for an OFDM system.

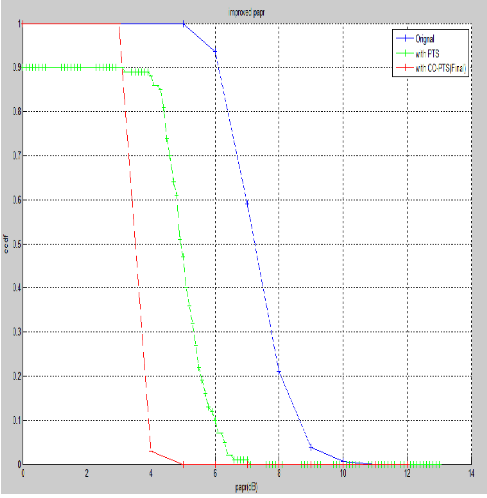

Fig.6: Plot PAPR Reduction according to ccdf before and after applies PTS and COPTS.

In Figure 6 show the comparison curve of peak average to power ratio according to ccdf by using different techniques. To calculate the PAPR reduction of an OFDM system according to complementary cumulative distribution function (ccdf) measurement after and before apply partial transmit sequence (PTS) and complexity PTS (COPTS). It is clearly seen at the comparison curve peak average to power ratio (PAPR) value according to complementary cumulative distribution function (ccdf) measurement vary from 10 to 11, 7 to 8 and 5 to 6 by using ordinary technique, partial transmit sequence (PTS) technique and reduced complexity partial transmit sequence (COPTS) technique respectively which is shown in Figure 6.

V. CONCLUSION

OFDM is a very attractive technique for multicarrier transmission and has become one of the standard choices for high – speed data transmission over a communication channel. It has various advantages; but also has one major drawback: it has a very high PAPR. In this dissertation, the different properties of an OFDM System are analyzed and the advantages and disadvantages of this system are understood. The bit error rate (BER) is also plotted against the signal to noise ratio (SNR) to understand the performance of the OFDM system. We have also aimed at investigating some of the techniques which are in common use to reduce the high PAPR of the system. Among the various techniques that we took up for study, but we found out or in others words using Partial Transmit Sequence (PTS) and Complexity PTS (COPTS).

Table 1: PAPR reduction according to different techniques

Various Techniques	BER (at 5dB)	PAPR Value Vary
Ordinary	0.011	10 to 11
PTS	0.014	7 to 8
COPTS	0.025	5 to 6

From the comparison curve of the PTS and COPTS techniques, we could infer that COPTS is more effective in PAPR reduction and it varies between 5 to 6 with the comparison of other techniques, such as Selected Mapping (SLM), Partial Transmit Sequence (PTS). Various PAPR reduction techniques have been developed with tradeoff between PAPR reduction capability and loss in data rate, degradation of BER performance, increase in signal power and increase in computational complexity. Modified PAPR reduction techniques with low computational complexity can be applied to high data rate OFDM Systems. Future studies may include a combination of different schemes.

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